Math 2A Quiz 1 Version 4
Fri Oct 7, 2016

NAME YOU ASKED TO BE CALLED IN CLASS:

SCORE: 30 **/ 30 POINTS** 24 + 6

1. **NO CALCULATORS OR NOTES ALLOWED**
2. **UNLESS STATED OTHERWISE, YOU MUST SIMPLIFY ALL ANSWERS**
3. **SHOW PROPER CALCULUS LEVEL WORK TO JUSTIFY YOUR ANSWERS**

Determine if $y = Ax + Be^{-2x} + \frac{x^2}{2}$ is a family of solutions of the DE $(2x+1)y'' + 4xy' - 4y = 4x^2 + 4x + 4$. **SCORE:** 6 **/ 6 PTS**

State your conclusion clearly.

$y' = A + -2Be^{-2x} + x$ 1

$y'' = 4Be^{-2x} + 1$ 1

$(2x+1)(4Be^{-2x}+1) + 4x(A + -2Be^{-2x} + x) - 4(Ax + Be^{-2x} + \frac{x^2}{2}) = 4x^2 + 4x + 4$

1 $\cancel{8xBe^{-2x}} + 2x + \cancel{4Be^{-2x}} + 1 + \cancel{4xA} + \cancel{-8xBe^{-2x}} + 4x^2 - \cancel{4Ax} - \cancel{4Be^{-2x}} - \frac{4x^2}{2} = 4x^2 + 4x + 4$

$2x^2 + 2x + 1 = 4x^2 + 4x + 4$

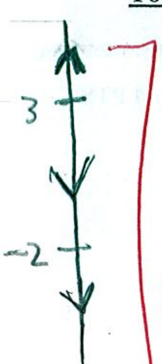
2

not a family of solutions

1

Consider the DE $\frac{dy}{dx} = (y^2 - y - 6)(y+2)$. **SCORE:** 6 **/ 6 PTS**

[a] Find all equilibrium solutions of the DE and classify each as stable, unstable or semi-stable.
You must draw a phase portrait to get full credit.



1

$\frac{dy}{dx} = (y-3)(y+2)(y+2)$

1 $y = 3$, unstable 1

1 $y = -2$, semistable 1

[b] If $y = m(x)$ is a solution of the DE such that $m(5) = 1$, what is $\lim_{x\to\infty} m(x)$?

$\lim_{x\to\infty} m(x) = -2$

1

Consider the IVP $y' = 2xy^2 - 3x$, $y(-1) = 2$. Use Euler's method with $h = 0.2$ to estimate $y(-0.6)$. **SCORE: 4 / 4 PTS**

$y_1 = y_0 + h(f(x,y))$

$y(-0.8) = 2 + 0.2((2(-1)(2)^2 - (3)(-1))$

$y(-0.8) = 2 + 0.2(-5)$ 1

$y(-0.8) = 1$ 1

$y(-0.6) = 1 + 0.2((2)(-0.8)(1)^2 - (3)(-0.8))$

$y(-0.6) = 1 + 0.2(-1.6 + 2.4)$

$y(-0.6) = 1 + 0.2(0.8)$ 1

$y(-0.6) = 1.16$ 1

In a certain society, the rate at which a person's wealth changes is proportional to the difference between their wealth and a fixed baseline (call it B, where $B > 0$). If everyone is getting poorer (except for those whose wealth equals the baseline), write a DE for the wealth of a person whose current wealth is half of the baseline. **SCORE: 4 / 4 PTS**

Justify the signs of all symbolic constants (other than B) in your DE properly, but briefly, as shown in lecture.
Do NOT use the absolute value function in your answer.

$A(0) = \frac{1}{2}B$

$\frac{dA}{dt} = K(A - B)$ 2

Since $A < B$, $A - B$ will be negative, therefore K is positive so that $\frac{dA}{dt}$ will be negative ½ 1 ½

What does the Existence and Uniqueness Theorem tell you about possible solutions to the IVP $(y')^3 - 1 = x + y$, $y(1) = -2$? **Justify your answer properly, but briefly.** **SCORE: 4 / 4 PTS**

$(y')^3 = 1 + x + y$

$y' = \sqrt[3]{1+x+y}$ 1 ← continuous on an interval where $x + y \neq -1$

$f_y = \frac{1}{3}(1+x+y)^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{(1+x+y)^2}}$ ← continuous on $x + y > -1$ 1

1 $y(1) = -2$ does not fall inside this interval since $1 + -2 = -1$

So E + U theorem does not tell us anything about the uniqueness of its solutions 1